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Symmetry restoration and tachyon condensation in open string theory

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Abstract: It has recently been argued that D-branes in bosonic string theory can be described as noncommutative solitons, outside whose core the tachyon is condensed to its ground state. We conjecture that, in addition, the local U(1) gauge symmetry is restored to a U(∞) symmetry in the vacuum outside this core. We present new solutions obeying this boundary condition. The tension of these solitons agrees exactly with the expected D-brane tension for arbitrary noncommutativity parameter θ, which effectively becomes a dynamical variable. The restored U(∞) eliminates unwanted extra modes which might otherwise appear outside the soliton core.

Keywords: Solitons Monopoles and Instantons, Bosonic Strings, D-branes.

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Contents

1. Introduction

The general theory of relativity follows largely from the demand that the laws of physics take the same form in all coordinate systems. In string theory, the massless boson associated to this coordinate invariance — namely the graviton — is just one mode of an infinite tower of mostly massive string states. Associated to this infinite tower of modes is a stringy generalization of coordinate invariance. In the usual perturbative string vacuum, almost all of the string modes are massive and almost all of this stringy symmetry is accordingly spontaneously broken [1].

One may expect that string theory itself largely follows from the demand of stringy symmetry. However, despite the spectacular developments of the last five years, the nature of this stringy symmetry remains enigmatic.[1] In this paper we investigate this issue of (open) stringy symmetry restoration in the context of a recent circle of ideas involving tachyon condensation, D-branes and noncommutative geometry. We will consider only the classical[2] open bosonic string.

[1] A recent discussion can be found in [2].

[2] Quantum effects could well be important in tachyon condensation, but we will not consider them.

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Following the work of Sen [3, 4], it is widely believed that the endpoint of the condensation of the open string tachyon is the closed string vacuum. There is by now compelling evidence for this conjecture from diverse points of view, including numerical computations [5, 6, 7] using Witten's open string field theory [8]. Moreover, Sen has argued [3] (see also [9, 10, 11]) that D-branes in bosonic string theory can be viewed as solitons of the open string tachyon. Outside the core of the soliton the tachyon is in its ground state, and the theory is in the closed string vacuum with no open string excitations.

Recently Harvey, Kraus, Larsen and Martinec [12] and Dasgupta, Mukhi and Rajesh [13] have shown that turning on a large B field enables an elegant realization of D-branes as tachyonic solitons. Techniques from noncommutative field theory [14] can be used to construct the D-brane soliton in the $\theta \to \infty$ limit of large noncommutativity. The soliton and D-brane tensions agree exactly in this limit. A simple and beautiful explanation of the non-abelian structure of D-branes is found [12], with a natural embedding into string field theory [15].

However, even with these improvements several puzzles remain. In order to eliminate unwanted propagating open string states far outside the D-brane soliton core (i.e. in the closed string vacuum), one must assume that the coefficients in the tachyon-Born-Infeld lagrangian take special values together with a special choice of field variables. Even with these assumptions, unwanted propagating modes persist inside the core in the bifundamental of $\mathrm{U}(N) \times \mathrm{U}(\infty - N)$, where N is the number of D-branes. Although plausible mechanisms [16, 17] for the elimination of these modes have been proposed, it is unsatisfying that these depend on unknown higher stringy corrections and cannot be seen directly from the lagrangian employed in the analysis. In addition it is difficult to understand why $1/\theta$ corrections would not spoil the exact agreement found in [12] between the soliton and D-brane tensions.

In this paper we consider the open bosonic string theory in the presence of a maximal rank B field. We propose that in the process of tachyon condensation, as the tachyon rolls to its minimum, the noncommutative gauge field simultaneously rolls to a maximally symmetric configuration, about which the noncommutative gauge symmetry is fully unbroken, and becomes a linearly realized $\mathrm{U}(\infty)$.[3] The propagation of open string modes in this 'nothing' state is forbidden by the $\mathrm{U}(\infty)$ symmetry, and there is no need to invoke higher-order stringy corrections or special values of coefficients for their elimination.

We also modify the proposed identification of D-branes as noncommutative tachyon solitons by demanding that far from the core of the soliton, the solution approaches the nothing state, in which the $\mathrm{U}(\infty)$ symmetry of the noncommutative field theory (which is broken to a local $\mathrm{U}(1)$ on the D-brane) is completely unbroken.

[3] We will refer to a configuration as having unbroken gauge symmetry if all fields are left invariant by the gauge transformations. Note that, with this usage, the usual perturbative vacuum of a gauge theory breaks local gauge invariance as $\delta A \neq 0$ for non-constant gauge transformations.

We construct exact soliton solutions of the noncommutative tachyon-Born-Infeld lagrangian obeying the modified boundary conditions, without expanding in $1/\theta$. It is further argued that these are exact-to-all-orders solutions of classical open string theory. The soliton tension exactly matches the expected D-brane tension. Furthermore, the propagation of open string modes far from the core, (i.e. in the nothing state) is forbidden as above by the $\mathrm{U}(\infty)$ symmetry. We regard these successes as evidence for the conjecture that the vacuum outside the D-brane core is the state of fully unbroken open string symmetries. On the other hand the situation for the bifundamentals is somewhat improved, but not fully resolved, as will be discussed in section 4.1.

One way of understanding the θ-independence of the D-brane tension is that, in the context of tachyon condensation, θ is effectively a dynamical variable. In a sense (to be made precise herein), our proposal is that θ effectively relaxes to ∞ at the boundary.

While some puzzles are resolved in our approach, a significant new puzzle arises. In addition to the solutions corresponding to D-branes, there are a number of other spurious solutions obeying the same boundary conditions for which we have no physical interpretation. These must be understood or somehow excluded before the picture presented here can be regarded as complete.

2. The action in shifted variables

2.1 The action

The euclidean action for $\mathrm{U}(1)$ open bosonic string theory contains the terms [18, 4] (see also [19, 20, 21])

$$S = \frac{1}{G_o^2 \alpha'^{13} (2\pi)^{25}} \int d^{26}x \left(V(T) \sqrt{\det(G + 2\pi\alpha' F)} + \frac{\alpha'}{2} f(T) D_\mu T D^\mu T \sqrt{\det G} + \cdots \right). \tag{2.1}$$

The tachyon potential V has a maximum at $T = T_{\rm max}$ corresponding to the unstable perturbative string vacuum and a minimum at $T = T_{\rm min}$ which should contain no perturbative open string excitations. According to [3] the minimum obeys

$$V(T_{\rm min}) = 0\,, \tag{2.2}$$

and the maximum is determined from the D25-brane tension to be, in our conventions

$$V(T_{\rm max}) = 1\,. \tag{2.3}$$

The universal coefficient of the potential term in (2.1) was demonstrated with worldsheet methods in [3]. In addition it has been conjectured that $f(T_{\rm min}) = 0$ [19, 20].[4] This will not play an essential role in our analysis, although it is required in [12].

[4] Note that if f is smooth at $T_{\rm min}$ it can in any case be set to one by a field redefinition.

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We wish to study the open bosonic string theory in the background of a constant B field. According to [22, 23], the euclidean action in this background continues to be given by (2.1), except that:

(a) space becomes noncommutative, i.e. all products in (2.1) are replaced by star products, with a noncommutativity tensor Θ, whose value is given below;

(b) the parameters that appear in (2.1); the open string metric $G_{\mu\nu}$, the noncommutativity tensor $\Theta^{\mu\nu}$ and the open string coupling G_o are related to closed string moduli by the formulae

$$\begin{aligned} 2\pi\alpha' G^{\mu\nu} + \Theta^{\mu\nu} &= \left(\frac{2\pi\alpha'}{g + 2\pi\alpha' B}\right)^{\mu\nu}, \\ G_o^2 &= g_{\rm str}\sqrt{\frac{\det(g + 2\pi\alpha' B)}{\det g}}\,. \end{aligned} \tag{2.4}$$

Here g and $g_{\rm str}$ are the usual constant closed string metric and coupling.

We are thus led to study a U(1) noncommutative gauge theory, interacting with a scalar field (the tachyon) that transforms in the adjoint of the gauge group. Note that we have not taken the $\alpha' \to 0$ scaling limit, so Born-Infeld corrections are retained. The noncommutative action (2.1) together with (2.4) is identical to that considered in [12] (prior to taking the $\theta \to \infty$ limit). An alternate form of the action, used for example in [13], [19]–[21] differs by higher derivative tachyon terms which would not affect our conclusions.

We choose $B_{\mu\nu}$ so that space is maximally noncommuting, i.e. Θ has maximal rank. We parameterize space with complex coordinates z^m, $m = 1, \ldots, 13$ obeying

$$[z^m, \bar{z}^{\bar{n}}] = i\Theta^{m\bar{n}}\,. \tag{2.5}$$

2.2 Brief review of the operator formalism

In this subsection we recall certain facts about noncommutative field theories, and especially noncommutative gauge theories, that we will use in our construction below. See, for instance, [14] for more details.

The algebra of functions on a 26 dimensional noncommutative space is represented by operators on the Hilbert space of a thirteen dimensional particle. On this space, we define thirteen annihilation operators $a_{\bar{m}}$ and an equal number of creation operators $a_m^\dagger$

$$a_{\bar{m}} = -i\Theta^{-1}_{\bar{m}n} z^n\,, \qquad a_m^\dagger = i\Theta^{-1}_{m\bar{n}} \bar{z}^{\bar{n}}\,. \tag{2.6}$$

These operators obey the commutation relations

$$[a_m^\dagger, a_{\bar{n}}] = -i\Theta^{-1}_{m\bar{n}}\,. \tag{2.7}$$

Several useful relations in translating from functions to operators are

$$\int d^{2n}x \to (2\pi)^n \sqrt{(-)^n \det\Theta}\,\mathrm{Tr}\,,$$
$$\partial_m \to -[a_m^\dagger,\ \ ],$$
$$\partial_{\bar m} \to [a_{\bar m},\ \ ]\,. \tag{2.8}$$

We now consider a noncommutative gauge theory written in the operator language. The covariant derivative of a field φ that transforms in the adjoint of the noncommutative gauge group may be cast in the form

$$D_m\varphi = \partial_m\varphi + i[A_m,\varphi] = -[C_m,\varphi]\,; \qquad D_{\bar m}\varphi = \partial_{\bar m}\varphi + i[A_{\bar m},\varphi] = [C_{\bar m},\varphi]\,, \tag{2.9}$$

where

$$C_m = -iA_m + a_m^\dagger\,, \qquad C_{\bar m} = iA_{\bar m} + a_{\bar m}\,. \tag{2.10}$$

The noncommutative field strength is

$$F_{m\bar n} = i[C,\bar C]_{m\bar n} - \Theta^{-1}_{m\bar n}\,, \tag{2.11}$$

where $[C,\bar C]_{m\bar n} = [C_m, C_{\bar n}]$ and $\Theta^{-1}_{m\bar n}\Theta^{\bar n p} = \delta_m{}^p$. The fields C_m, $C_{\bar m}$ transform homogeneously under gauge transformations. In particular, the field configurations $C_m = C_{\bar m} = 0$ leave the gauge symmetry unbroken.

2.3 The action and equations of motion

The noncommutative action (2.1) (for open string modes in the presence of a $B_{\mu\nu}$ field) can be rewritten in operator language as

$$S = \frac{\sqrt{-\det\Theta}}{G_o^2\alpha'^{13}(2\pi)^{12}}\mathrm{Tr}\Big[V(T)\sqrt{\det(G + 2\pi\alpha'(i[C,\bar C] - \Theta^{-1}))} + \\ + \alpha' f(T)[C_p,T][T,C^p]\sqrt{\det(G)} + \cdots\Big]\,. \tag{2.12}$$

Operators in (2.12) are appropriately ordered so as to reproduce string amplitudes; the precise ordering of operators in this action will not be important for us.

The tachyon equation of motion that follows from (2.12) is

$$2\alpha' f(T)[C_m,[C^m,T]]\sqrt{\det G} - \alpha' f'(T)[C_m,T][C^m,T] + V'(T)\sqrt{\det M} = 0\,, \tag{2.13}$$

where we have defined

$$M = G + 2\pi\alpha'(i[C,\bar C] - \Theta^{-1})\,. \tag{2.14}$$

The equation for C_m is

$$-\frac{1}{2}\alpha'[T,[C^m,T]f(T)]\sqrt{\det G} + i\pi\alpha'[C_{\bar n}, V(T)\sqrt{\det M}(M^{-1})^{m\bar n}] = 0\,. \tag{2.15}$$

3. The nothing state

In the variables (2.10), the usual vacuum with a single D25-brane is

$$T = T_{\max}\,, \qquad C_m = a_m^\dagger\,. \tag{3.1}$$

In [4] it was conjectured that the 'nothing' state with no D25-branes is

$$T = T_{\min}\,, \qquad C_m = a_m^\dagger\,. \tag{3.2}$$

We would like to propose instead that the nothing state is

$$T = T_{\min}\,, \qquad C_m = 0\,. \tag{3.3}$$

Due to (2.2), (3.3) and (3.2) are energetically degenerate.

Under a local U(1) gauge transformation

$$\delta A_m = \partial_m \epsilon + i[A_m, \epsilon]\,, \tag{3.4}$$

it follows from (2.7) that C_m transforms as

$$\delta C_m = i[C_m, \epsilon]\,. \tag{3.5}$$

Hence, as remarked above, the nothing state (3.3) is fully invariant under this symmetry. The local U(1) symmetry is restored to an unbroken U(∞) symmetry of unitary transformations on the quantum mechanical Hilbert space.

We will now argue that fluctuations about the fully symmetric state (3.3) have no perturbative propagating open string degrees of freedom. (2.1) describes a noncommutative gauge theory whose matter fields all transform in the adjoint of the gauge group. Gauge invariance dictates that derivatives and gauge fields A appear in the action only in the combination C_m. Thus fluctuations about any background with $C_m = 0$ are governed by an action with no derivatives beyond those that appear in the star product. In particular, quadratic terms in the action have no derivatives (as the star product acts trivially on such terms); this statement is unchanged by nonlinear field redefinitions. Further, U(∞) invariance ensures that explicit derivative terms are not dynamically generated. Thus, perturbatively, open string modes do not propagate about the background (3.3).

3.1 Nothing in ordinary variables

In [23] it was shown that there is a nonlocal field redefinition which relates the noncommutative field strength F to an 'ordinary' field strength, which we shall denote $F^{\rm ord}$, appearing in the commutative formulation of the same theory. Under this Seiberg-Witten map [23], a constant noncommutative field F maps to a constant ordinary field strength, whose value is given by

$$F^{\rm ord} = F\frac{1}{1+\Theta F}\,. \tag{3.6}$$

In (3.3) the noncommutative field strength F takes the constant value $-\Theta^{-1}$, and so corresponds to a divergent ordinary field strength.[5] Schematically, (3.3) in ordinary variables, and the gauge $B=0$ takes the form

$$T = T_{\min} \qquad F^{\mathrm{ord}}_{\mu\nu} = \infty\,. \tag{3.7}$$

By the second equation in (3.7) we mean that, in the vacuum state, F^{ord} is a rank 26 tensor, all of whose eigenvalues diverge.

Thus the conjecture of this section may be worded in ordinary variables as follows: the perturbative open string vacuum state with $T = T_{\max}$, and a finite constant F^{ord} (in the gauge $B=0$) is unstable and decays to the nothing state, $T = T_{\min}$ and infinite F^{ord}. This is in part possible because at $T = T_{\min}$, $V(T)$ vanishes and hence there is no energy cost to changing F^{ord}.

3.2 Nothing as $\Theta = \infty$

We have argued above that in ordinary variables the nothing state is given by (3.7), or equivalently by

$$T = T_{\min}\,, \qquad F^{\mathrm{ord}}_{\mu\nu} = 0\,, \qquad B_{\mu\nu} \to \infty\,. \tag{3.8}$$

In order to analyze this state, we move to yet another set of variables; the gauge field whose noncommutativity is set by the large B of (3.8). In terms of the new noncommutative F (whose background value is zero in the nothing state), the action takes the form (2.1) with parameters

$$G_{\mu\nu} = -\left(b\frac{1}{g}b\right)_{\mu\nu}, \qquad \Theta^{\mu\nu} = 2\pi\alpha'\left(\frac{1}{b}\right)^{\mu\nu}, \qquad G_o^2 = g_{\mathrm{str}}\sqrt{\frac{\det b}{\det g}}\,, \tag{3.9}$$

where $b_{\mu\nu} = 2\pi\alpha' B_{\mu\nu}$. Notice that $\Theta^2 \equiv \mathrm{Tr}(\Theta G \Theta G) = (2\pi\alpha')^2 \mathrm{Tr}(\frac{1}{g}b\frac{1}{g}b) \to \infty$ in the limit under consideration. Thus, focusing on energies for which noncommutative phases are finite, explicit derivatives in the action (2.1) may be dropped. Defining a rescaled gauge field $H_\mu = g_{\mu\alpha}(1/b)^{\alpha\nu}A_\nu$ (2.1) takes the form

$$S = \frac{1}{G_o^2 \alpha'^{13}(2\pi)^{25}} \int d^{26}x \sqrt{\det G}\Bigg(V(T)\sqrt{\det(\delta_\mu^\nu + 2\pi i\alpha'[H_\mu, H_\alpha]g^{\alpha\nu})} + \\ + \frac{\alpha' f(T)}{2}[H_\mu, T][T, H_\nu]g^{\mu\nu} + \cdots\Bigg)\,. \tag{3.10}$$

In the operator language

$$S = \frac{2\pi}{g_{\mathrm{str}}}\mathrm{Tr}\left(V(T)\sqrt{\det(\delta_\mu^\nu + 2\pi i\alpha'[H_\mu, H_\alpha]g^{\alpha\nu})} + \frac{\alpha' f(T)}{2}[H_\mu, T][T, H_\nu]g^{\mu\nu} + \cdots\right), \tag{3.11}$$

[5] We are grateful to Jeff Harvey for explaining this point.

where we have used

$$\frac{\sqrt{\det\Theta}\sqrt{\det G}}{(2\pi)^{13}\alpha'^{13}G_o^2} = \frac{1}{g_{\rm str}}\,. \tag{3.12}$$

Thus fluctuations about the nothing state are governed by the action (3.11), the dimensional reduction of the infinite N open string field theory to a spacetime point. Note that B does not enter into (3.11), consistent with the expectation that the end product of tachyon condensation is insensitive to the initial value of Θ.

4. D23 Branes

4.1 The soliton solution

We wish to find a soliton solution which is translationally invariant in 24 directions and approaches the nothing state in the complex transverse z^1 direction away from the core. For these purposes we take

$$\begin{aligned} G^{1\bar{i}} = \Theta^{1\bar{i}} &= 0\,, \qquad i = 2,\ldots,13\,, \\ \Theta^{1\bar{1}} &= \theta G^{1\bar{1}}\,. \end{aligned} \tag{4.1}$$

Consider the field configuration

$$\begin{aligned} T - T_{\rm min} &= (T_{\rm max} - T_{\rm min})P_{N_1}\,, \\ C_i &= P_{N_2}a_i^\dagger\,, \qquad i = 2,\ldots,13\,, \quad C_1 = 0\,, \end{aligned} \tag{4.2}$$

where P_{N_k} is a rank N_k projection operator in the Hilbert space constructed from $a_1^\dagger$. For example we could take P_{N_k} to be the projection onto the first N_k states of the harmonic oscillator. Then the right hand side of (4.2) vanishes exponentially outside the soliton core, and the solution is asymptotic to the nothing state (3.3).[6] (In contrast, the approximate solutions found in [13, 12] have the same tachyon field but $C_m = a_m^\dagger$, and are asymptotic to (3.2).)

It is easy to check that (4.2) solves the equations of motion (2.13), (2.15). The first term in the tachyon equation (2.13) vanishes if we require

$$[P_{N_1}, P_{N_2}] = 0\,. \tag{4.3}$$

The second term vanishes because $V'(T_{\rm max}) = 0$. (4.3) also implies the separate vanishing of both terms in the C^m equation (2.15).

Of course, the true equations of motion that follow from the action (2.12) have an infinite number of terms (from the $\cdots$ in (2.12)) that we have not considered here. However, each of these terms contains at least one factor of a covariant derivative

[6] We note that in the $\theta \to \infty$ limit this solution is of the general form required for the string field theory construction described in [15].

of either T or F. Since all covariant derivatives of T and F given in (4.2) vanish, additional terms in the equation of motion also vanish to all orders in the α' expansion. Non-perturbative effects could alter the situation. It is rather surprising that an exact-to-all-orders solution can be constructed without even knowing what the lagrangian is! Usually such constructions are possible only with supersymmetry: here it is a consequence of the magic of noncommutativity.

4.2 The soliton action

We will interpret solutions of the form (4.2) with

$$P_{N_1} = P_{N_2} = P_N \tag{4.4}$$

as N coincident D-branes. Solutions of the lagrangian (2.1) with $P_{N_1} \neq P_{N_2}$ certainly exist but they do not correspond to conventional D-branes (the spectrum is wrong). The role of these solutions - or a rationale for their exclusion - must be understood before the picture presented here can be regarded as satisfactory. For now we consider (4.4). Using (4.1), (2.12) reduces to

$$S = \frac{V(T_{\max})\mathrm{Tr}P_N}{G_o^2 \alpha'^{13}(2\pi)^{12}} \sqrt{-\det\Theta}\sqrt{\det G}\sqrt{1+(\frac{2\pi\alpha'}{\theta})^2}. \tag{4.5}$$

We wish to rewrite this in terms of the coupling (G'_o), measure ($\sqrt{\det G'}$) and noncommutativity parameter ($\Theta'^{i\bar{j}}$) with respect to the 24 longitudinal dimensions. It follows from (2.4) that these are related to the 26-dimensional quantities by

$$\begin{aligned} G_o^2 &= G_o'^2 \sqrt{1+\left(\frac{\theta}{2\pi\alpha'}\right)^2}\,, \\ \sqrt{\det G} &= G_{1\bar{1}}\sqrt{\det G'}\,, \\ \sqrt{-\det\Theta} &= \theta G^{1\bar{1}}\sqrt{\det\Theta'}\,. \end{aligned} \tag{4.6}$$

The trace gives

$$\mathrm{Tr}P_N = \frac{NV_{24}}{\sqrt{\det G'}(2\pi)^{12}\sqrt{\det\Theta'}}\,, \tag{4.7}$$

where $V_{24} = \int d^{24}y\sqrt{\det G'}$. Substituting into (4.5) and using (2.3) yields

$$S = \frac{NV_{24}}{G_o'^2\alpha'^{12}(2\pi)^{23}}\,. \tag{4.8}$$

All θ dependence has disappeared, and this is exactly the action of N parallel D23-branes.

4.3 The spectrum

We now describe the spectrum of the solution (4.2) and (4.4). We choose a basis in which

$$P_N = \sum_{a=1}^{N} |a\rangle\langle a| \,. \tag{4.9}$$

U(N) **Adjoint fields.** U(N) adjoint fluctuations in the tachyon field can be expanded as

$$\delta T = \sum_{a,b=1}^{N} T_{ab}(y)|a\rangle\langle b| \,, \tag{4.10}$$

where y is a longitudinal 24-dimensional coordinate and T_{ab} is hermitian. As in [12], substituting into (2.12) reveals 24-dimensional tachyons in the adjoint of U(N). A similar expansion gives U(N) gauge fields. This is exactly the low-lying spectrum of N bosonic D23-branes. Higher mass open string states on the D25-brane similarly descend to adjoint fields on the D23-branes, as in [12, 15].

U($\infty - N$) **Adjoint fields.** Derivative terms in modes of the form $T_{jk}(y)|j\rangle\langle k| + h.c.$, where $j, k > N$ are projected out of the quadratic action because C is proportional to P_N. Hence there are no propagating adjoint U($\infty - N$) fields. In [12, 13] the gauge field does not have a transverse profile (as is consistent with the boundary condition (3.2)) and C is proportional to the identity instead of P_N. In order to eliminate propagation of these modes, the additional assumption $f(T_{\rm min}) = 0$ is required. Even then, if f is quadratic or otherwise smooth about the minimum it may be set to one with a field redefinition. In these variables - which are the natural ones for studying propagation - propagating U($\infty - N$) tachyons reappear. In any case, with the solution (4.2) the absence of such propagating modes is a natural consequence of the symmetries and no such additional assumptions about the f prefactor or restrictions on field variables are necessary.

U($\infty - N$) $\times$ U(N) **Bifundamental fields.** We may also consider U($\infty - N$) $\times$ U(N) bifundamental modes of the form $T_{ak}(y)|a\rangle\langle k| + h.c.$ where $a \leq N, k > N$.[7] Again, because of the projection operators in C, these modes do not acquire ordinary kinetic terms. They do however have a nonvanishing quadratic action involving fixed matrices. Substituting into (2.12) we get

$$S_{\rm eff}(T_{ak}) \sim {\rm Tr}\left[a^j a_j^\dagger T_{ak}^2\right] \qquad (j = 2, \ldots, 13) \,. \tag{4.11}$$

[7] In fact these modes can be gauged away or are eaten by the Higgs mechanism [12], but then similar comments pertain to the fluctuations of the gauge field. We consider the tachyon here for notational simplicity.

Eq. (4.11) is the action for a charged particle in a magnetic field of strength $1/\theta$. It has a discrete spectrum with spacing of order $1/\theta$, rather than a spectra of continuous momenta. In particular, there are no bifundamental excitations with energies below $1/\theta$.

Formally these modes disappear as the longitudinal noncommutativity θ is taken to zero, however higher order corrections to the action (2.1) appear to be suppressed by powers of α'/θ, and hence cannot be ignored at small θ. Hence we cannot make firm conclusions about the spectrum at small θ.

5. Discussion

In this paper we have proposed answers to two puzzles relating to the condensation of the open bosonic string tachyon (in the presence of a B field)

(a) Why are there no open string excitations at the bottom of the tachyon well for any value of Θ ?

(b) Why is the condensed state at the bottom of the well independent of Θ?

We propose that as the tachyon rolls to its minimum, the gauge field also dynamically rolls to its maximally symmetric value (with nonzero field strength), and the fully unbroken gauge invariance prohibits perturbative propagation. This rolling is in part possible because, exactly at the bottom of the tachyon well, the coefficient of the Born-Infeld term in the action (2.1) vanishes, and there is no energy cost for changing a constant field strength. Using the Seiberg-Witten change of variables, this maximally symmetric configuration with nonzero field strength and finite Θ can be reinterpreted as one with zero field strength and $\Theta = \infty$.

Restated, we propose that Θ (as set by the value of the commutative $\mathcal{F} = F^{\mathrm{ord}} + B$ at infinity) is effectively a dynamical variable that, regardless of its initial value, rolls to infinity in the process of tachyon condensation. The Θ independence of the tension and spectrum of our soliton is a consequence of this dynamical nature of Θ. If this proposal is indeed correct, it would be very interesting to understand in detail the dynamics that sends Θ to infinity, rather than any other (seemingly degenerate) value, as the tachyon rolls to its minimum.

In closing we note that the $\mathrm{U}(\infty)$ symmetry restoration described here is obviously closely related to the symmetry restoration in the cubic formulation [24] of Witten's open string field theory. It would be of interest to understand this connection in more detail.

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